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Finite Difference Solution of Free Convective Heat Transfer of Non-Newtonian Power Law Fluids from a Vertical Plate

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Abstract : A laminar free convective flow of a power fluid over a vertical plate is investigated. The equations governing the fluid flow are solved numerically using an implicit finite difference scheme which is shown to be unconditionally stable. The effects of the flow parameters on the velocity field and temperature profile are reflected through graphs.

Keywords : Power law fluid, magnetic field, Reynold number, Prandtl number.

1. Introduction

Although natural convection has been of interest for a number of years most of the studies published, so fare have been concerned with Newtonian rather non-Newtonian liquids. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate. Because of the application of non-Newtonian fluids in industries processing in power law fluids by free convection along a vertical plate has been investigated by many researchers. The fundamental case of laminar natural convection heat transfer from an isothermal vertical plate to a power law fluid was analysed by Acrivos (1960). Hence investigation concerning combined thermal and species diffusion driven flows were carried out by several Researchers such as Sommer 1956 Wilcox [1957], Emery et al [1971], Som and Chen [1984]. Huang, et al [1990] obtained the local similarity solutions of free convection heat transfer from a vertical plate to non-Newtonian power law fluids. All these workers obtained the solution methods, integral methods or numerical methods temperature using asymptotic methods, integral methods or numerical methods. Padhy and Pattnayak [1997] have studied the mass transfer and free convective effects of a power law fluids past an impulsively started vertical plate. Anwar Hassain and Wilson [2002] discussed the natural convection flow in a fluid saturated porous medium enclosed by non-isothermal walls with heat generation. Similarly Nasser [2008] studied the problem of unsteady free convection with heat and mass transfer from an isothermal vertical flat plate to a non-Newtonian power law fluid immersed in a saturated porous medium and also Olajuwon [2008] investigated the flow and convection heat transfer in a pseudo plastic power law fluid past a vertical plate with heat generation. Chamkha Aly and Mansour [2010] presented in tabular and graphical form to show the effects of material parameters of the problem on the solution. Thus the aim of the present study is to investigate laminar free convective flow of power law fluid over a vertical plate in presence of magnetic field. The interaction of magnetic field is proved to be counter productive in enhancing velocity distribution. The effect of different flow parameters on velocity and temperature are reflected in the figures.

2. Formulation of the Problem

We consider free convective heat transfer of a power law fluid X' - axis is taken along the infinite vertical plate in the upward direction of y' -axis is taken normal to it. The plate starts moving impulsively in the upward direction with constant velocity U_0. An oscillating temperature is applied on the plate and a foreign mass is injected into the fluid. Then the physical qualities are function of y' and t' only. In the power law model for non-Newtonian fluids, the convective laminar and steady conservation equations can be written as

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial x'}+v'\frac{\partial v'}{\partial y'}=g\beta(T'-T'_\infty)+\frac{k}{\rho}\frac{\partial}{\partial y'}\left|\frac{\partial u'}{\partial y'}\right|^{n-1}\frac{\partial u'}{\partial y'}-\frac{\sigma B_0^2 u'}{\rho} \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\alpha\left(\frac{\partial^2 T'}{\partial y'^2}\right) \tag{3}$$

where g is acceleration due to gravity, α represents the thermal diffusivity, β is coefficient of thermal expansion of fluid, *k* is fluid consistency index for power law fluid, ρ is fluid density, *n* is power law index u', v' are stream wise and transverse velocity respectively. Similarly x' and y' are stream wise and transverse co-ordinate. T' is temperature of the fluid and t' is time.

The initial boundary conditions are

$t'=0, u'=v'=0, T'=T'_\infty \quad \forall x'$ and y'

$t'>0, x'=0, u'=0, T'=T'_\infty$

$t'>0, u'=v'=0, T'=T'_\infty$ at $y'=0, x'>0$

$$t'>0, u'=0, T'=T'_\infty \text{ at } y'\to\infty, x'>0 \tag{4}$$

Introducing the following non-dimensional qualities

$x'=xL,\ y'=yL, u'=Uu\ , v'=Uv$ and $T'=T_\infty+T(T_w-T_\infty)$

the model equations with initial and boundary reduces to the following non-dimensional from

$$y=\frac{y'}{v'_0 r}\ ,\ x=\frac{x'}{v'_0 r}\ , u=\frac{u'}{v'_0 r}, v=\frac{v'}{v'_0 r}$$

$$T=\frac{T-T'_\infty}{T'_w-T'_\infty}$$

$Gr=g\beta(T'_w-T'_\infty)r/v'_0$ (Grashof number)

$P_r=\dfrac{v'_0 r}{\alpha}$, Prandtl number

$M^2=\dfrac{\sigma B_0^2 u'}{\rho}$ = Hartmann number

where $r=\left[\dfrac{k}{\rho {v'_0}^2}\right]^{\frac{1}{n}}$

Using the non-dimensional quality the equations (1), (2), (3) boundary conditions (4) respectively

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)+G_r T-M^2 u \tag{6}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Re\,P_r}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

$t=0, u=v=0, T=0 \qquad \forall x$ and y

$t>0, u=v,=0 \quad$ at $\; x=0$

$t>0, u=v=0, T=1$ at $\; y=0,\; x>0$

$$t>0, u=0, T=0 \text{ at } y\to\infty, x>0 \tag{8}$$

where $Gr=\dfrac{g\beta L(T_w-T_\infty)}{U^2}$ is the Grashof number, $M^2=\dfrac{\sigma B_0^2 L}{\rho U}$ is the Hartmann number, $Re=\dfrac{\rho UL}{\mu}$ is the Reynold's number and $P_r=\dfrac{c_p vp}{k}$ is the Prandtl number. Here the characteristic velocity U is assumed as $\left(\dfrac{\rho L^n}{k}\right)^{1/(n-2)}$, $C_f=$ Skin Friction : $\left(\dfrac{\partial u}{\partial y}\right)^n \quad y=0$

$N_{ux}=$ Local Nusselt Number : $-\dfrac{\partial T}{\partial y}$ at $y=0$

3. Method of Solution

The equation (5)-(8) are solved by implicit finite difference method. For discretisation in space and time a uniform mesh of space step h and time Δt are employed so that the grid points are $(y_i, t_i)=(ih, j\Delta t) i=0,1,2,....,N, j=0,1,2,\ldots,$ M where $tm=M\Delta t$, is the time at which velocity stress and temperature are computed. We choose N so that the boundary condition at ∞ holds at $(Nh, j\Delta t)$. The discredited form of equations (1), (2) and (3) respectively obtained as

$$\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}+\frac{v_{i,j+1}-v_{i,j-1}}{2\Delta y}=0 \tag{9}$$

$$\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta t}+u_{ij}\left(\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}\right)+v_{ij}\left(\frac{v_{i,j+1}-v_{i,j-1}}{2\Delta y}\right)$$

$$=G_r T_{i,j}+n\left[\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}\right]^{n-1}-\left[\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2}\right]-\;M^2 u_{ij} \tag{10}$$

$$\frac{T_{i+1,j}-T_{i-1,j}}{2\Delta t}+P_r\left[u_{ij}\left(\frac{T_{i+1,j}-T_{i-1,j}}{2\Delta x}\right)+v_{ij}\left(\frac{T_{i,j+1}-T_{i,j-1}}{2\Delta y}\right)\right]$$

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i,j-1}}{(\Delta y)^2}\,{}_{\substack{i=1,2,\ldots,N\\ j=1,2,\ldots,M}}\;. \tag{11}$$

The above discretised equation (9) to (11) are solved iteratively using the following algorithm.

Step 1: Initialise $u_{i,j}^{(0)}, v_{i,j}^{(0)}, T_{i,j}^{(0)}$, for all (i, j)

Step II

For $k = 0,1,2,3 \ldots K$ max

For $j = 1,2,3 \ldots M$

For $i = 1,2,3\ldots N$

Compute

$$u_{i,j}^{(k+1)}=\frac{\left[\left(\frac{u_{i-1,j}-u_{i+1,j}}{2\Delta t}\right)-v_{i,j}\left(\frac{v_{i,j+1}^{(k)}-v_{i,j-1}^{(k+1)}+G_rT_{ij}^{(k+1)}}{2\Delta y}\right)\right]}{\left[u_{i+1,j}^{(k)}-u_{i-1,j}^{(k+1)}\right]+\frac{2n}{(\Delta y)^2}\left|\frac{u_{i,j+1}^{(k)}-u_{i,j-1}^{(k+1)}}{2\Delta y)}\right|^{n-1}}$$

$$\frac{\dfrac{+n\left|u_{i,j+1}^{(k)}-u_{i,j-1}^{(k+1)}\right|^{n-1}\times\left\{\dfrac{u_{i,j+1}^{(k)}+u_{i,j-1}^{(k+1)}}{(\Delta y)^2}\right\}}{2\Delta y}}{+M^2} \tag{12}$$

Step-**III** If $j=11, i=1,2,\ldots N$

Compute

$$v_{i,j}^{(k+1)}=\Delta y\left[\frac{u_{i,j}^{(k+1)}-u_{i+1,j}^{(k)}}{\Delta x}\right]+v_{i,j-1}^{(k+1)} \tag{13}$$

Else for $j=1$, $i=1,2,\ldots N$

Compute

$$v_{i,j}^{(k+1)}=\frac{2\Delta y}{3}\left[\frac{u_{i-1,j}^{(k+1)}-u_{i+1,j}^{(k)}}{2\Delta x}\right]+\frac{4}{3}v_{i,j-1}^{(k)}-\frac{1}{3}v_{i,j-1}^{(k)} \tag{14}$$

Step-**IV** Compute

$$T_{i,j}^{(k+1)} = \frac{(\Delta y)^2}{-2}\left\{\left(\frac{T_{i+1,j}^{(k)} - T_{i-1,j}^{(k+1)}}{2\Delta x}\right) + P_r\left[u_{i,j}^{(k+1)}\left(\frac{T_{i+1,j}^{(k)} - T_{i-1,j}^{(k+1)}}{2\Delta x}\right)\right.\right.$$

$$\left.\left. + v_{i,j}^{(k+1)}\left(\frac{T_{i,j+1}^{(k)} - T_{i,j-1}^{(k)}}{2\Delta y}\right) - \left(\frac{T_{i,j+1}^{(k)} + T_{i,j-1}^{(k)}}{(\Delta y)^2}\right)\right]\right\} \quad (15)$$

Repeat the steps (2) to (4) until the relative errors of two consecutive values of $u_{i,j} v_{i,j}$ and $T_{i,j}$ are less than a given tolerance.

4. Results and Discussion

We have solved the governing equation 5-7 using the boundary conditions 8 using finite difference method which explain briefly in the previous section. The velocity and temperature profile for different parameters are shown through Fig.1-2. The velocity and temperature profile for Reynolds number is shown in Fig.1. Reynolds number is the ratio between viscous and inertia term. Viscosity of the fluid increases with increase of the Reynolds number which decrease the velocity (see Fig. 1(a)). With increase of the velocity, the fluid thickness as shown in Fig. 1(b).

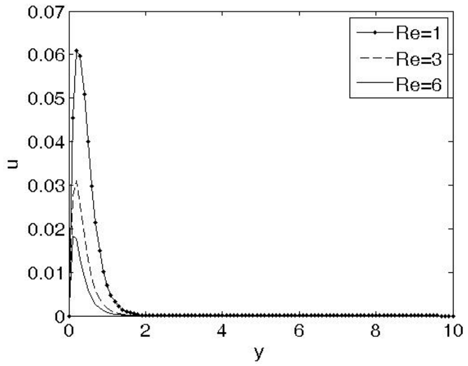

Fig. 1(a)

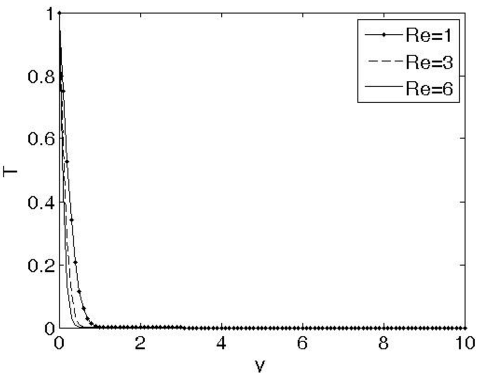

Fig. 1(b)

The velocity and temperature of the fluid for different Prandtl number is shown in Fig.2. Prandtl number increase the viscous diffusivity of the fluid at the surface which enhanced the velocity of the fluid near the surface which depicted by Fig.2(a). This is inversely proportional to thermal diffusivity which is the reason behind the increase of thermal boundary layer thickness.

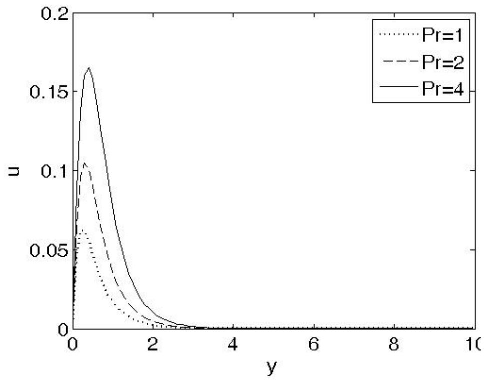

Fig. 2(a)

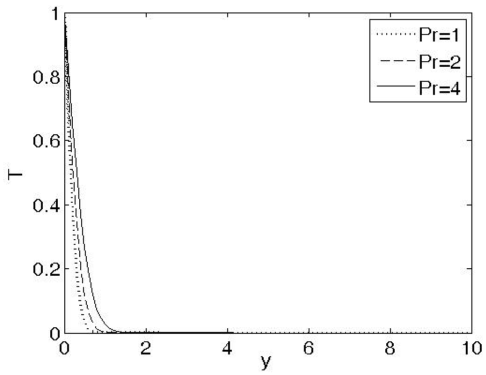

Fig. 2(b)

The velocity highly influenced by the other parameter as magnetic parameter, Grashof number and power-law index which shown through Fig.3-5. Transverse magnetic creates a Lorentz drag force which resist the flow velocity of the fluid. As a result the momentum boundary layer thickness decrease with increase of the magnetic parameter (see Fig. 3). Viscous force acting on the fluid is inversely proportional to the Grashof number and so the velocity of the fluid increases with increase of the Grashof number. It is interested to note that the velocity of the fluid is lower for the case of Newtonian fluid (n = 1) while for $n \neq 1$, the velocity of the fluid increase i.e., for both the pseudoplastic and dilutant fluid, the velocity is more than the Newtonian fluid (see Fig.5).

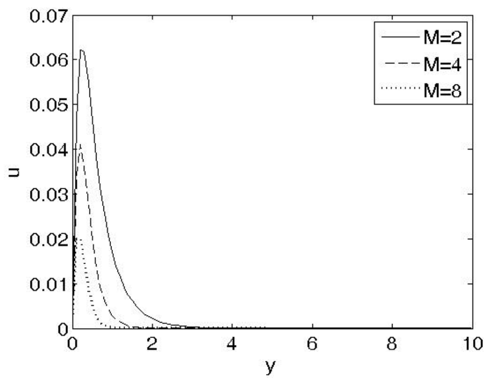

Fig. 3

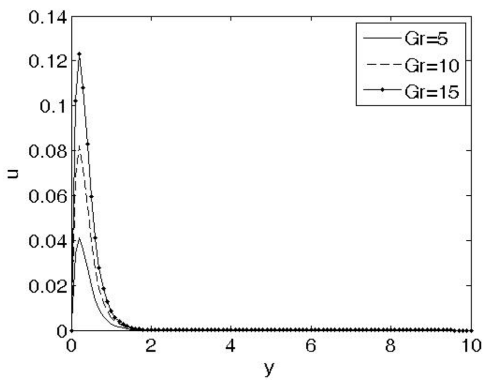

Fig. 4

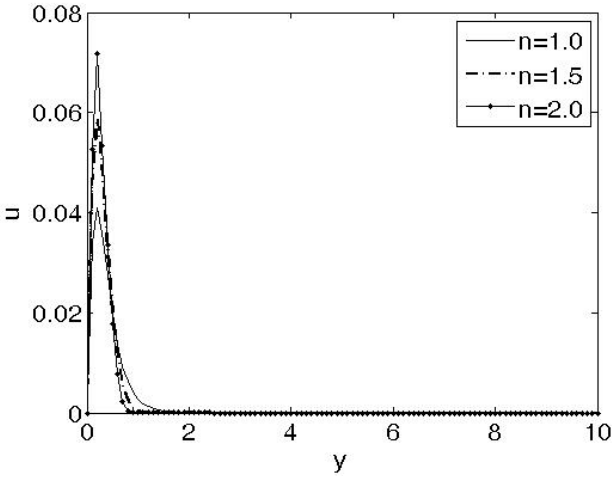

Fig. 5

Table 1. The effect of all the parameter on skin friction and Nusselt number

M	n	Pr	Re	Gr	Cr	Nu
2	1	1	1	5	0.5535	0.3857
4					0.1746	0.3857
8					0.0468	0.3857
2	15	1	1	5	0.4618	0.3857
	2				0.3924	0.3857
2	1	2	1	5	0.6771	0.2775
		4			0.7748	0.1979
2	1	1	3	5	0.3266	0.6122
			6		0.2016	0.7529
2	1	1	1	10	1.1070	0.3857
				15	1.6593	0.3861

For the physical interest view, we find the influence of magnetic parameter, power-law index, Prandtl number, Reynolds number and Grashof number on the skin friction and local Nusselt number is shown by Table 1. It is interested to note that magnetic parameter decreases the velocity of the fluid which helps to reduce the skin friction at the surface. While the local heat transfer rate not influenced by the magnetic parameter. The skin friction at the surface decreases when the fluid changes from dilutant to pseudoplastic.

5. Conclusion

In the present work we have considered the free convective heat transfer of Non-newtonian power law fluid, in presence of magnetic field. The system of nonlinear equations obtained by discretisation is solved numerically by finite difference method. The solution obtained well agree with the Newtonian case. It gives improved result taken into consideration the behaviour of magnetic field. One of the major observation in the present investigation is that the velocity decreases with an increase magnetic field, this method well for other Non-newtonian fluid flow parameters.

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